BIRZEIT UNIVERSITY

$\frac{84}{100}$

Mathematics Department

First Hour Exam /summer 2015

Math 132

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$\frac{25}{16} - \sinh^2 = 1$

$\sinh^2 x = \frac{25}{16} - \frac{16}{16} = \frac{9}{16}$

$= \frac{3}{4}$

Q#1 80% Circle the correct answer.

1. If $\cosh x = \frac{5}{4}$, x>0 then sinh x =

a) $\frac{3}{5}$ b) $\frac{3}{4}$ c) $\frac{4}{5}$ d) $\frac{5}{4}$

2. The order of the functions x^x, e^x, $\sqrt{x}$, logx from slower growing to fastest growing as $x \to \infty$

a) x^x, e^x, $\sqrt{x}$, logx b) $\sqrt{x}$, logx, x^x, e^x

c) logx, $\sqrt{x}$, x^x, e^x d) logx, $\sqrt{x}$, e^x, x^x

$\lim_{x \to \infty} \frac{x^x}{e^x}$

$\left(\frac{x}{e}\right)^x$

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3. $\int_{\ln 2}^{\ln 4} \frac{e^x}{e^x+1} dx =$

a) $\ln 2$ b) $\ln 3$ c) $\ln 4$ d) $\ln 5 - \ln 3$

$f(x) = \ln \frac{x^x}{e^x}$

$\ln$

$\frac{e^x \cdot e^{-x}}{(e^x+1)\, e^{-x}}$

$\frac{1}{1+e^{-1}}$

$\frac{1}{x}$

$\int_{\ln 2}^{\ln 4} \ln|e^x+1|$

$\left[\ln e^{\ln 4}+1\right] - \left[\ln e^{\ln 2}+1\right]$

$\ln 4 - \ln 2$

$\ln \frac{4}{2} = \ln 2$

4. if $y = (\ln x)^x$ then $\frac{dy}{dx}$

a) $(\ln x)^x [\frac{1}{\ln x} + \ln(\ln x)]$ b) $(\ln x)^{x-1}$

c) $(\ln x)^{x-1}$ d) $(\ln x)^x (\frac{2\ln x}{x})$

$\ln y = \ln(\ln x)^x$

$\frac{y'}{y} = x \ln(\ln x)$

$\cancel{x} \cdot \frac{\frac{1}{\cancel{x}}}{\ln x} + \ln(\ln x) \cdot 1$

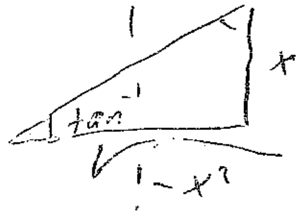
$\frac{1}{\ln x} + \ln(\ln x)\Big] (\ln x)^x$

1

$\ln\left(\frac{1}{\sqrt{3}}\right)$ $\frac{1}{3}+\frac{3}{3}=\frac{4}{3}$

$-\ln 2$

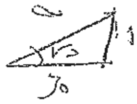
$\ln 4-\ln 3-\ln 2$



5. $\int_{?}^{\frac{1}{\sqrt{3}}}\frac{dx}{(1+x^2)\tan^{-1}x}=$

a) $\ln 4-\ln 3$ b) $\ln 2-\ln 3$

c) $2\ln\sqrt{3}$ d) $\frac{\pi}{12}$



$\ln|u|$

$\ln\frac{\pi}{8}-\ln\frac{\pi}{4}$ $\ln\frac{4}{6}$

6. $\int\frac{dx}{(x-1)\sqrt{x^2-2x}}$

a) $\sqrt{x^2-x}+c$ b) $\frac{1}{2}\ln\left|x^2-2x\right|+c$

c) $\sec^{-1}\left|x-1\right|+c$ d) $\sinh^{-1}(x-1)+c$

$\frac{du}{u\sqrt{u^2-1}}=$

$(x^2-2x+1)-1$

$(x-1)(x-1)$

$\sqrt{(x-1)^2-1}$

$u=x-1$

$du=dx$

$\sec^{-1}|u$

$\int\frac{du}{u\sqrt{u^2-1}}$

$\sec^{-1}|u^2-1|$

$(x-1)^2$

x^2-2x

7. $\int_{-1}^{0}\frac{dx}{x^2+2x+2}=$

a) 1 b) $\frac{1}{2}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

$\frac{1}{(x+1)(x+1)+2=1}$

$\frac{1}{(x^2+2x+1)+1}$

$(x+1)(x+1)$

$(x+1)^2+1$

$u=x+1$

$du=dx$

$\int\frac{1}{u^2+1}$

$\tan^{-1}|u|$

$\tan^{-1}(x+1)$

$\tan(1)-\tan(0)$

$\frac{\pi}{4}-0$

8. $\int x^2e^{3x}dx=$

a) $\frac{e^{3x}}{3}(9x^2+6x+2)+c$ b) $e^{3x}(\frac{1}{3}x^2-\frac{2}{9}x+\frac{2}{27})+c$

c) $\frac{e^{3x}}{27}(9x^2+6x-2)+c$ d) $\frac{e^{3x}}{3}(9x^2-3x+2)+c$

x^2 e^{3x}

$2x$ $\frac{e^{3x}}{3}$

2 $\frac{e^{3x}}{9}$

0 $\frac{e^{3x}}{27}$

$e^{3x}\left(\frac{1}{3}x^2-\frac{2}{9}x+\frac{2}{…}\right.$

$x^2\cdot\frac{e^{3x}}{3}+2x\cdot\frac{e^{3x}}{9}+\frac{2e^{3x}}{27}$

9. $\lim_{x\to 0}\frac{\sinh\ 2x}{\sin\ x}=$

a) 2 b) 1 c) $\frac{1}{2}$ d) doesnt exist

$\frac{e^{x}+e^{-x}}{2}\cdot 2$ $\frac{\cosh 2x\cdot 2}{\cos x}$

$e^{0}-e^{0}=1-1=0$

$e^{?}+3x+5=6$

$e^{x}+3x=1$

$\boxed{x=0}$

$f^{-1}(6)=\frac{1}{f'(f^{-1}(6))}$

$\frac{1}{f'(0)}$

$e^{0}+3=1+3 = \frac{1}{4}$

10. If $f(x)=e^{x}+3x+5$ then $(f^{-1})'(6)=$

a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{1}{e+3}$ d) 3

$4\times4\times4\times4$

$\frac{2\times2}{4}\times\frac{2\times2}{4}\times\frac{2\times2}{4}$

$\frac{\ln 32}{\ln 4}$

$\frac{\ln 2^5}{\ln 2^2}$ $\frac{5}{2}$

11. $\log_4 32$

a) 2 b) $\frac{1}{2}$ c) $\frac{5}{4}$ d) $\frac{5}{2}$

$f(x)=6$ $x=0$

$6=e^{x}+3x+5$

$\ln y=\sin x\ln 2$

$\frac{y'}{y}=\ln 2\cos x\cdot y$

$y'=\ln\cos\pi\cdot 2^{\sin\pi}$

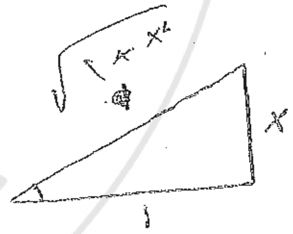
$\ln 2\cdot 2^{\sin\pi}$ 2^{0} 1

12. if $y=2^{\sin x}$ then $\frac{dy}{dx}$ when $x=\pi$ is:

a) 0 b) 1 c) $\ln 2$ d) $-\ln 2$

13. if $y=\tan^{-1}(\frac{1}{x})$ then $\sin y=$

a) $\frac{1}{\sqrt{1+x^2}}$ b) $\frac{\sqrt{1+x^2}}{x}$ c) $\frac{x}{\sqrt{1+x^2}}$ d) $\frac{1}{\sqrt{1-x^2}}$



14. $\int_0^1 \frac{1}{\sqrt{x}+x\sqrt{x}}dx$

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

$\frac{1}{\sqrt{x}+x\sqrt{x}}$

$u=\sqrt{x}$

$du=\frac{1}{2\sqrt{x}}$ $du\,2\sqrt{x}=dx$

$\frac{du\cdot 2u^2}{u+u^2\cdot u}$ du 1

$\frac{1}{u+xu}$

$2\int\frac{u^2}{u+u^3}$

$du\cdot 2\sqrt{x}=dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} \Rightarrow du \cdot 2\sqrt{x} = dx$

$\frac{\cosh u}{u} \cdot 2u \, du$

$2 \cosh u \cdot u$

15. $\int_0^1 \frac{\cosh(\sqrt{x})}{\sqrt{x}} dx =$

a) $\frac{(e-1)^2}{2}$ b) $2\sinh 1$ c) $2(\cosh 1) - 2$ d) $\frac{e}{2}$

$u \quad \cosh u$

$0 \quad \sinh u$

$2 \cdot u \sinh u$

$2\sqrt{1} \sinh\sqrt{1} - 20$

$2 si$

16. $\int \frac{dx}{\sqrt{2x - x^2}} =$

a) $2\sqrt{2x - x^2} + c$ b) $\sin^{-1}(\frac{x-1}{2}) + c$

c) $\sin^{-1}(x-1) + c$ d) $\sec^{-1}(x-1) + c$

$-(x^2 - 2x + 1 - 1)$

$-((x-1)^2 + 1)$

$1 - (x-1)^2$

$\frac{1}{\sqrt{1-u^2}}$

17. $\int \tan^{-1} x \, dx =$

a) $x \tan^{-1} x + \sqrt{1+x^2} + c$ b) $x \tan^{-1} x - \ln\sqrt{1+x^2} + c$

c) $x \tan^{-1} x - \sqrt{1+x^2} + c$ d) $x \tan^{-1} x - 2\sqrt{1+x^2} + c$

$x = a \tan\theta$

$\frac{x}{a} = \tan\theta$

$\tan^{-1}$

18. If $f(x) = \frac{x+1}{x-2}$ then

a) $f^{-1}(x) = \frac{2x+1}{x-1}$ b) $f^{-1}(x) = \frac{x-1}{x+2}$

c) $f^{-1}(x) = \frac{2x+1}{x+2}$ d) $f^{-1}(x) = \frac{2x-1}{x+2}$

$y(x-2) = (x+1)$

$f^{-1}(x), f(\frac{1}{x})$

19. $4^{\ln 2} =$

a) 2 b) 4

c) $2\ln 2$ d) $\ln 4$

$2 - x$

$2^{2\ln 2} = 2^{\ln 4}$

$f(f^{-1}(x)) = x$

20. $\int_1^{\sqrt{e}} \frac{dx}{x\sqrt{1-(\ln x)^2}} =$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$

c) $\frac{\pi}{2}$ d) π

$\sec^{-1}(\ln x)$

$\sec^{-1}(\frac{1}{2}) - \sec^{-1}(1)$

$x + 1 - 2 + 2$

$3 + 2y - 2$

$\frac{2y+1}{y-1}$

$\frac{\cancel{x-2}}{\cancel{x-2}} + \frac{3}{x-2}$

$y - 1 = \frac{3}{x-2}$

$x - 2 = \frac{3}{y-1} + \frac{2(y-1)}{y-1}$

$y = 1 + \frac{3}{x-2}$ 4

$\frac{1}{\cos c}$

$y - 1 = x - 2$

$\frac{\pi}{6} - \frac{\pi^3}{3x^2}$ $4 + 1$

$\frac{1}{\cos 0}$

$2\frac{\pi}{6} = \frac{\pi}{3}$

Q#2 20% Evaluate.

1. $\int \ln(x^2+1)\,dx$

$u = \ln(x^2+1) \qquad dv = dx$

$du = \dfrac{2x}{x^2+1} \qquad v = x$

~~$u = x^2+1$~~

~~$\dfrac{du}{2x} = \dfrac{2x\,dx}{2x}$~~

~~$\int \dfrac{\ln u\,du}{2x}$~~

$x\ln|x^2+1| - \int \dfrac{2x^2}{x^2+1}$

~~$x\ln|x^2+1| - 2\int \dfrac{x^2}{x^2(1+\frac{1}{x^2})}$~~

$u = x^2+1$

$du = 2x$

~~$\int \dfrac{2x^2}{u}\cdot\dfrac{du}{2x} \qquad \dfrac{2}{1+x^{-2}}$~~

$x\ln|x^2+1| - \int \dfrac{2x^2}{x^2+1}$

$u = 1 - x$

$du = -2x$

$\frac{\sqrt{u}}{-2x\ x^2}$

$|1-x|(1+x)$

$\sqrt{1-x^2} \cdot \frac{\sqrt{1+x^2}}{1+\sqrt{x}}$

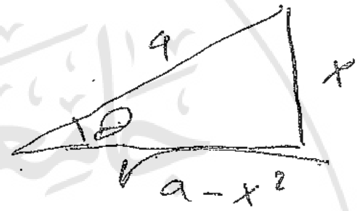
$\overline{1-x^2}$

$$2.\int \frac{\sqrt{1-x^2}}{x^2}dx$$

~~$\sin$~~

$x = a\sin\theta$

$x^2 = a^2 \sin^2\theta$



$a\cos\theta = \frac{\sqrt{a-x^2}}{a}$

$dx = \cos\theta\, d\theta$

$\int \frac{\cos\theta}{\sin^2\theta} \cdot \cos\theta\, d\theta$

$\int \sec^2 - 1$

$\tan -$

$\int \frac{\cos^2\theta}{\sin^2\theta}$

$\int \cot^2\theta\, d\theta$

$\frac{1}{\tan}$

$\int \csc^2\theta - 1\, d\theta$

10

$-\cot\theta - \theta + C$

$-\frac{\sqrt{a-x^2}}{x} - \sin^{-1}(x) + C$